Examiners' Report:
Final Honour School of Mathematics Part A
Michaelmas Term 2020

March 3, 2021

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1.

Table 1: Numbers in each class

Range	Numbers					Percentages %				
	2020	2019	2018	2017	2016	2020	2019	2018	2017	2016
70–100	43	57	57	57	50	32.58	35.19	35.62	36.77	34.97
60–69	65	71	69	62	63	49.24	43.83	43.12	40	44.06
50–59	21	27	22	31	26	15.91	16.67	13.75	20	18.18
40–49	3	5	9	4	3	2.27	3.09	5.62	2.58	2.1
30–39	0	1	3	1	0	0	0.62	1.88	0.65	0
0–29	0	1	0	0	1	0	0.62	0	0	0.7
Total	132	162	160	155	143	100	100	100	100	100

- **Numbers of vivas and effects of vivas on classes of result.**
  Not applicable.
- **Marking of scripts.**
  All scripts were single marked according to a pre-agreed marking scheme which was strictly adhered to. The raw marks for paper A2 are out of 100, and for the other papers out of 50. For details of the extensive checking process, see Part II, Section A.
- **Numbers taking each paper.**
  All 132 candidates are required to offer the core papers A0, A1, A2 and ASO, and five of the optional papers A3-A11. Statistics for these papers are shown in Table 2 on page 2.

Table 2: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
A0	132	32.55	7.57	66.81	11.64
A1	132	35.41	6.89	66.1	9.44
A2	132	69.89	13.28	65.76	11.38
A3	66	39.61	5.68	65.76	10
A4	96	32.2	5.64	67.03	8.39
A5	62	38.9	5.73	66.39	10.77
A6	76	33.05	9.14	65.84	12.7
A7	56	34.64	5.48	67.59	9.36
A8	122	35.47	5.67	66.16	8.67
A9	77	33.38	6.1	64.19	9.3
A10	42	38.69	6.02	65.81	9.81
A11	70	39.89	7.66	68.99	15.04
ASO	132	36.44	6.51	66.82	11.92

B. New examining methods and procedures

In light of Covid 19, the department took steps to mitigate the impact of the pandemic. This included changing the examinations to an open-book versions of the standard exam papers and delaying Part A examinations so that they took place in Michaelmas term 2020. An additional 30 minutes(with the exception of A2 which had 1 hour) was added on to the exam duration to allow candidate the technical time to download and submit their examination papers via Weblearn.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

Due to the uncertainty with the pandemic, the department decided that exams will be taken online for Trinity Term 2021.

D. Notice of examination conventions for candidates

The first notice to candidates was issued on 26th February 2020 and the second notice on the 8th September 2020.

These can be found at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/ba-master-mathematics/examinations-assessments/examination-20, and contain details of the examinations and assessments. The course handbook contains the link to the full examination conventions and all candidates are issued with this at induction in their first year. All notices and examination conventions are on-line at
https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. General Comments on the Examination

Acknowledgements

- Nicole Collins and Barbara Galinska for their work in supporting the Part A examinations throughout the year, and for her help with various enquiries throughout the year.
- Waldemar Schlackow for running the database and the algorithms that generate the final marks, without which the process could not operate.
- Charlotte Turner-Smith for her help and support, together with the Academic Administration Team, with marks entry, script checking, and much vital behind-the-scenes work.
- The assessors who set their questions promptly, provided clear model solutions, took care with checking and marking them, and met their deadlines, thus making the examiners' jobs that much easier.
- Several members of the Faculty who agreed to help the committee in the work of checking the papers set by the assessors.
- The internal examiners and assessors would like to thank the external examiners, Jelena Grbic and Demetrios Papageorgiou, for helpful feedback and much hard work throughout the year, and for the important work they did in Oxford in examining scripts and contributing to the decisions of the committee.

Timetable

The examinations began on Monday 5th October and ended on Friday 16th October.

Mitigating Circumstances Notices to Examiners

A subset of the examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate. See Section E for further details.

Setting and checking of papers and marks processing

As is usual practice, questions for the core papers A0, A1 and A2 were set by the examiners. Usually these papers are also marked by the examiners but this year the marking period was October-November 2020 which coincided with teaching duties during term time. A decision was made to employ extra assessors from the Postgraduate Associates at the Department who marked of some of the questions for A0, A1 and A2 following the solutions and marking scheme provided by the examiners. The papers A3-A11, as well as each individual question on ASO, were set and marked by the course lecturers/assessors. The setters produced model answers and marking schemes led by instructions from Teaching Committee in order to minimize the need for recalibration.

The internal examiners met in December to consider the questions for Michaelmas Term courses (A0, A1, A2 and A11). The course lecturers for the core papers were invited to comment on the notation used and more generally on the appropriateness of the questions. Corrections and modifications were agreed by the internal examiners and the revised questions were sent to the external examiners.

In a second meeting the internal examiners discussed the comments of the external examiners and made further adjustments before finalising the questions. The same cycle was repeated in Hilary term for the Hilary term long option courses and at the end of Hilary and beginning of Trinity term for the short option courses. *Papers A8 and A9 are prepared by the Department of Statistics and jointly considered in Trinity term.* Due to Pandemic, the papers went through a second review to ensure questions would be suitable for Open book exams. Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.

Candidates accessed and downloaded their exam papers via the Weblearn system at the designated exam time. Exam responses were uploaded to Weblearn and made available to the Exam Board Administrator 48 hours after the exam paper had finished.

The process for Marking, marks processing and checking was adjusted accordingly to fit in with the online exam responses.

Assessors were provided with the electronic mark sheets and had 4 weeks to mark the scripts and return the marksheets to their dropbox in secure weblearn. A team of graduate checkers under the supervision of Elle Styler met virtually to script check the papers assigned to them. This included cross-checking the marksheets for each candidate against the mark scheme to spot any unmarked questions or part of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the marks scheme, noting any incorrect addition. An examiner was present at all times to authorise any required corrections.

Determination of University Standardised Marks

The examiners followed the standard procedure for converting raw marks to University Standardized Marks (USM). The raw marks are totals of marks on each question, the USMs are statements of the quality of marks on a standard scale. The Part A examination is not classified but notionally 70 corresponds to 'first class', 50 to 'second class' and 40 to 'third class'. In order to map the raw marks to USMs in a way that respects the qualitative descriptors of each class the standard procedure has been to use a piecewise linear map. It starts from the assumption that the majority of scripts for a paper will fall in the USM range 57-72, which is just below the II(i)/II(ii) borderline and just above the I/II(i) borderline respectively. In this range the map is taken to have a constant gradient and is determined by the parameters C_1 and C_2, that are the raw marks corresponding to a USM of 72 and 57 respectively. The guidance requires that the examiners should use the entire range of USMs. Our procedure interpolates the map linearly from $(C_1, 72)$ to $(M, 100)$ where M is the maximum possible raw mark. In order to allow for judging the position of the II(i)/III borderline on each paper, which corresponds to a USM of 40, the map is interpolated linearly between $(C_3, 37)$ and $(C_2, 57)$ and then again between $(0, 0)$ and $(C_3, 37)$. It is important that the positions of the corners in the piecewise linear map are not on the class borderlines in order to avoid distortion of the class boundaries. Thus, the conversion is fixed by the choice of the three parameters

C_1, C_2 and C_3, the raw marks that are mapped to USM of 72, 57 and 37 respectively.

The examiners chose the values of the parameters as listed in Table 3 guided by the advice from the Teaching Committee and by examining individuals on each paper around the borderlines.

Table 3: Parameter Values

Paper	C1	C2	C3
A0	(37.8,72)	(24.3,57)	(13.96,37)
A1	(42,72)	(27,57)	(15.51,37)
A2	(79.8,72)	(58.8,57)	(33.78,37)
A3	(45.6,72)	(33.6,57)	(19.30,37)
A4	(36.6,72)	(24.6,57)	(14.13,37)
A5	(43,70)	(32.9,57)	(18.9,37)
A6	(40.2,72)	(23.7,57)	(13.61,37)
A7	(38.4,72)	(27.9,57)	(16.03,37)
A8	(40.6,72)	(28.6,57)	(16.43,37)
A9	(39.6,72)	(27.6,57)	(15.86,37)
A10	(44.6,72)	(32.6,57)	(18.73,37)
A11	(45,70)	(32,57)	(18.38,37)
ASO	(41.8,72)	(28.3,57)	(16.26,37)

Table 4 gives the resulting final rank and percentage of candidates with this overall average USM (or greater).

Table 4: Rank and percentage of candidates with this overall average USM (or greater)

Av USM	Rank	Candidates with this USM or above	%
87.7	1	1	0.76
84.9	2	2	1.52
84.35	3	3	2.27
83.4	4	4	3.03
81.9	5	5	3.79
79.5	6	6	4.55
79.3	7	7	5.3
79.2	8	8	6.06
78.8	9	9	6.82
78.4	10	10	7.58
77.9	11	11	8.33
76.9	12	12	9.09
76.8	13	14	10.61
76	15	17	12.88
75.5	18	18	13.64
75.4	19	19	14.39
75.3	20	21	15.91
75.2	22	22	16.67
74.5	23	23	17.42
74.1	24	24	18.18
74	25	25	18.94
73.3	26	26	19.7
72.9	27	27	20.45
71.8	28	28	21.21
71.6	29	29	21.97
71.4	30	30	22.73
71.3	31	32	24.24
71.15	33	33	25
71.1	34	34	25.76
71	35	35	26.52
70.5	36	36	27.27
70.3	37	37	28.03
70.1	38	38	28.79
70	39	41	31.06
69.6	42	42	31.82
69.5	43	43	32.58
69.4	44	44	33.33
69.3	45	45	34.09
68.6	46	47	35.61
68.3	48	49	37.12
68.1	50	50	37.88
68	51	53	40.15

Table 4: Rank and percentage of candidates with this overall average USM (or greater) [continued]

Av USM	Rank	Candidates with this USM or above	%
67.9	54	56	42.42
67.6	57	58	43.94
67.5	59	59	44.7
67.4	60	61	46.21
67.3	62	64	48.48
67.2	65	65	49.24
67.1	66	66	50
67	67	67	50.76
66.7	68	68	51.52
66.6	69	69	52.27
66.3	70	70	53.03
66.2	71	71	53.79
66.1	72	73	55.3
65.9	74	75	56.82
65.8	76	76	57.58
65.4	77	77	58.33
65.2	78	79	59.85
65.1	80	81	61.36
65	82	82	62.12
64.6	83	83	62.88
64.3	84	84	63.64
63.3	85	85	64.39
63.1	86	86	65.15
63	87	88	66.67
62.9	89	89	67.42
62.6	90	90	68.18
62.5	91	91	68.94
62.3	92	92	69.7
62.1	93	93	70.45
61.9	94	94	71.21
61.8	95	95	71.97
61.4	96	97	73.48
61.3	98	99	75
61.2	100	100	75.76
60.9	101	101	76.52
60.8	102	102	77.27
60.5	103	103	78.03
60.3	104	104	78.79
60.2	105	105	79.55
60	106	107	81.06
59.5	108	108	81.82
58.5	109	111	84.09

Table 4: Rank and percentage of candidates with this overall average USM (or greater) [continued]

Av USM	Rank	Candidates with this USM or above	%
58	112	112	84.85
57.5	113	114	86.36
57.2	115	115	87.12
56.8	116	116	87.88
56.1	117	117	88.64
55.95	118	118	89.39
55.8	119	119	90.15
55.7	120	120	90.91
55.2	121	121	91.67
53.9	122	122	92.42
53.6	123	123	93.18
53.4	124	124	93.94
53	125	125	94.7
52.7	126	126	95.45
52.2	127	127	96.21
52.1	128	128	96.97
50.6	129	129	97.73
48.7	130	130	98.48
48.3	131	131	99.24
45.6	132	132	100

Recommendations for Next Year's Examiners and Teaching Committee

Cross sectional paper: The examiners noted that the ASO paper remains difficult to scale yet different standards of questions remain an issue in particular it was highlighted that the following two subjects (Special Relativity and Mathematical Modelling in Biology) should be reviewed by Teaching Committee due to low numbers and mathematical content on the syllabus.

Naming conventions on exam papers: The examiners noted that there is a need for improved procedure to keep track of the right papers- with the date and name of the version in the file name. Further communications to setters required to highlight the importance of checking the final Copy of the exam papers.

B. Equality and Diversity issues and breakdown of the results by gender

Table 5, page 9 shows percentages of male and female candidates for each class of the degree.

Table 5: Breakdown of results by gender

Class	Number								
	2020			2019			2018		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
70–100	10	33	43 12	45	57	9	48	57	
60–69	22	43	65	28	43	71	19	50	69
50–59	7	14	21	12	15	27	8	14	22
40–49	2	1	3	2	3	5	3	6	9
30–39	0	0	0	0	1	1	1	2	3
0–29	0	0	0	0	1	1	0	0	0
Total	41	91	132	54	108	162	40	120	160
Class	Percentage								
	2020			2019			2018		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
70–100	24.39	36.26	30.32	22.22	41.67	35.19	22.5	40	35.62
60–69	53.66	47.25	50.45	51.85	39.81	43.83	47.5	41.67	43.12
50–59	17.07	15.38	16.22	22.22	13.89	16.67	20	11.67	13.75
40–49	4.88	1.1	2.99	3.7	2.78	3.09	7.5	5	5.62
30–39	0	0	0	0	0.93	0.62	2.5	1.67	1.88
0–29	0	0	0	0	0.93	0.62	0	0	0
Total	100	100	100	100	100	100	100	100	100

C. Detailed numbers on candidates' performance in each part of the exam

Individual question statistics for Mathematics candidates are shown in the tables below.

Paper A0: Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.91	15.91	4.41	117.00	0
Q2	18.35	18.35	3.53	117.00	0
Q3	9.39	9.60	4.18	30.00	1

Paper A1: Differential Equations 1

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.33	19.33	3.28	130.00	0
Q2	15.92	15.92	4.59	83.00	0
Q3	16.31	16.47	5.27	51.00	1

Paper A2: Metric Spaces and Complex Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.31	16.31	4.63	88.00	0
Q2	17.89	17.89	3.86	106.00	0
Q3	15.98	15.98	5.36	48.00	0
Q4	16.68	16.75	4.42	102.00	1
Q5	20.05	20.05	3.51	113.00	0
Q6	15.99	16.25	6.64	71.00	2

Paper A3: Rings and Modules

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.44	19.44	3.36	64.00	0
Q2	19.85	19.85	3.29	60.00	0
Q3	22.38	22.38	2.39	8.00	0

Paper A4: Integration

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.38	16.49	3.99	85.00	1
Q2	11.84	12.06	3.53	18.00	1
Q3	16.54	16.54	3.40	89.00	0

Paper A5: Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.59	18.59	2.97	56.00	0
Q2	21.00	21.00	3.58	56.00	0
Q3	16.25	16.25	3.25	12.00	0

Paper A6: Differential Equations 2

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.13	17.29	5.11	66.00	2
Q2	14.53	14.77	6.18	48.00	1
Q3	17.42	17.42	4.94	38.00	0

Paper A7: Numerical Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.00	19.00	2.69	56.00	0
Q2	15.55	15.55	4.90	44.00	0
Q3	16.00	16.00	4.65	12.00	0

Paper A8: Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.91	18.91	3.12	102.00	0
Q2	17.27	17.31	3.82	55.00	1
Q3	16.47	16.62	3.20	87.00	2

Paper A9: Statistics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.25	15.25	4.06	40.00	0
Q2	17.46	17.46	3.75	56.00	0
Q3	16.93	16.93	2.82	58.00	0

Paper A10: Fluids and Waves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.28	19.28	4.14	40.00	0
Q2	19.65	19.65	4.25	31.00	0
Q3	18.85	18.85	2.19	13.00	0

Paper A11: Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	22.52	22.52	3.09	69.00	0
Q2	14.19	14.76	7.55	29.00	2
Q3	19.29	19.29	5.28	42.00	0

Paper ASO: Short Options

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.91	17.91	3.83	47.00	0
Q2	18.03	18.03	4.13	35.00	0
Q3	20.33	20.33	3.87	9.00	0
Q4	12.67	12.67	3.92	12.00	0
Q5	17.06	17.38	5.52	47.00	1
Q6	18.47	18.47	2.74	55.00	0
Q7	17.31	17.13	3.38	15.00	1
Q9	20.73	20.73	3.88	44.00	0

D. Comments on papers and on individual questions

The following comments were submitted by the assessors.

Core Papers

A0: Algebra 1

Question 1: Almost all candidates attempted this question with good results.

In 1a(ii) some candidates assumed incorrectly that all eigenspaces of a diagonalisable linear map must be one-dimensional. The same error was made in 1(b) and lead to the wrong conclusion that that $\ker(D - \mu I)$ is one-dimensional which simplifies the argument considerably.

Part(c) had only few correct arguments.

A frequent error in 1(c) was to treat 1(c) as a subcase of 1(b) i.e. assume wrongly that $T - \lambda I$ must be nilpotent in which case the statement of 1(c) is an immediate corollary of 1(b).

Some candidates claimed and used that D and N must be polynomials in T, which is correct but no proof was offered.

Almost all candidates used implicitly that D and N must preserve the generalised eigenspaces of T without proof.

Question 2:

Most students attempted this question, and generally the grades were quite high.

2a i) No significant issues

2a ii) No significant issues

2a iii) Most solved this correctly, about half and half using the two methods mentioned in the model solution.

2a iv) very few students solved this question correctly (under a fifth).

2b i) This was mostly solved correctly.

2b ii) and iii) many students did not solve these questions. In those that did the most common error was not carefully defining an inner product space. e.g. taking the space of all sequences without regard to convergence of the sum.

Question 3:

Very few students attempted question 3, and of those that did, generally the grades were quite low.

The main error which ran through all parts of the question, was assuming that the eigenvalues of A are only ± 1 (following 3a i)).

Hardly any students solved b ii) and iii). Most attempts at solutions had little logic to them.

A1: Differential Equations 1

Question 1: All but two of the candidates attempted this question. There were relatively few low scoring answers, and just over half the candidates scored at least 20 marks. In part (a) a surprising number of candidates did not seem to realise that the expressions in (2) are linear. In part (b) most errors were made whilst attempting to classify the critical points. Those who were able to complete part (c) often did so by applying the Bendixson-Dulac theorem with a constant Dulac function.

Question 2: This question was very popular, and most students managed to get a decent mark. Sadly, a lot of issues have to do with lack of proof reading, which leads to careless sign errors, or wrong integration (sometimes very bad ones, like $\frac{dz}{dt} = z^2$ gives $z = t^3 \ldots$ [*sic*]). Writing full sentences also seems too difficult, but I guess that might be too much to ask (especially for Q2a). A lot of students struggled a bit with the book keeping for Q2b, to find z. But the most difficult part seems to have been finding the domain of definition, only 2 students got it right (two!). Some of the students thought of checking the Jacobian, fewer checked that z does not blow up by checking its denominator, but failed to see the issues on $-1/2 < x < 0$ and $y^2 < 1 + 2x$: they just exclude the parabola from the domain of definition. This could suggest a lack of deep understanding of the concept of data curve and characteristics... Then Q2c was rather successful too, most students suggest splitting the domain in two on $[0, \pi/4[$ and $]\pi/4, \pi/2]$, some explaining a bit better than others why.

Question 3: Candidates did very well in the bookwork part of the question. Those who worked with admissible characteristic variables were usually able to determine the canonical form in b.(i), although some candidates did not obtain the correct result in the end due to miscalculation. In b.(ii) a few candidates verified that each function of the given shape solves the equation in question when it was asked to show that, conversely, each solution must be of that shape. Most candidates who successfully answered the question up to this point were then able to ascertain the exact values of f and g under the given boundary conditions, although there were quite a few small mistakes such as missing square roots or absolute values. In part c) of the question several candidates realized that the same characteristic variables employed in b) could be used to good effect there as well, leading to an equation for which the maximum principle was significantly simpler to prove. However, many candidates either omitted details about how to obtain the transformed equation, erroneously stating that the result had been obtained in b.(i) (NB: the two equations are not the same), or else did not carefully argue why the change of variables must preserve the boundary.

A2: Metric Spaces and Complex Analysis

Question 1. Bookwork seemed fine although a bunch of students accidentally defined uniform continuity. Part b) iii) confused plenty of people who got the implications mixed up. They thought that $\rho < d$ implies $B(x, r)$ in the ρ-metric would be contained in $B(x, r)$ in the d-metric. One wrinkle in the question is that if you observe that the balls in one metric are precisely balls in the other metric, then the question is pretty much immediate. In Part c) students were far happier making sense of the examples in iv) than the reverse direction in iii).

Question 2. This question was the more popular of the first two questions. Intuition in b) seemed fine so long as they had correctly understood the subspaces. The phrase "brief justification" was frequently treated as a license not to give an explanation based on definitions and ideas in the course. Plenty of students made the mistake of thinking the converse of b)i) was true and then went on to use this false implication in ii) and iii) which would be a mini disaster. The reverse direction of d) seemed like a heavy split between people who knew exactly what to do, and people who didn't even try or weren't even sure about how to go about proving it.

Question 3. This question seems to have been seen as one of the harder options with only around 1/3 candidates choosing to answer it. Proving the uniform convergence of infinite sums of functions may have put many off. Part (a) was well answered by almost everyone. Part (b) (i) caused some issues; proving that the limit function f satisfied $\int_\gamma f(z)dz = 0$ for every closed curve γ was generally well done, but it was very common to forget to verify that f is continuous so that Morera's Theorem applies. Part (b) (ii) was generally answered well with missing the $k!$ term being the only similar error from multiple candidates. Part (b) (iii) proved quite a mixed bag. Very few candidates realised that the first part was a consequence of the second part. A lot of candidates correctly prove the first part but failed to realise the one change in the argument required to prove the second. This appeared to be the result of a lack of understanding of what was required in order to establish uniform convergence, i.e. not realising the comparison bound needed to be independent of z_0. There were also some issues with using the reverse triangle inequality to obtain an upper bound for $|z - z_0|^{-(k+1)}$. Part (c) was correctly approached by the majority via comparing the integrals over the unit circle $\gamma(t) = e^{2\pi it}$ for $t \in [0, 2\pi)$, though some provided poor explanations for why any polynomial will integrate to 0 over this curve. A common error was a failure to realise that the sequence of polynomials need not have fixed degree; resulting in some incorrect arguments based on the existence of some $k_0 \in N$ such that for every polynomial p_n in the sequence, the k^{th} order derivative $p_n^{(k)} \equiv 0$ whenever $k \geq k_0$. Part (d) proved challenging for most candidates. Some used the integral test to prove convergence without any consideration of uniformity. Candidates that invoked the M-test generally did well, though there were several poor explanations of why proving uniform convergence on $\{z \in C : \Re(z) \geq 1 + de\}$ for $de > 0$ was sufficient. Another common issue was the failure to provide a statement of the M-test, despite the wording of the hint.

Question 4. The first few parts were mostly done well, although in part (c) most people did not distinguish between an essential singularity and a non-isolated singularity. There was a small typo in part (d) of the question, but most candidates did not spot this and allowance was made in marking. In part (d)(i), many people gave a correct calculation with no justification. Part (d)(ii) seemed to be found more difficult, but most candidates who thought to use Laurent's Theorem did well on this bit.

Question 5. The mostly computational aspect of this question proved attractive to candidates with around 5/6 choosing to answer it. The hidden theoretical aspect of Part (d) did however cause issues for a lot of candidates. Part (a) was answered correctly by almost every candidate. Part (b) resulted in a split; the residue was generally well defined by every candidate, but stating Cauchy's Residue Theorem caused a few issues. Those giving the simpler version involving a *positively oriented* curve were fine. However, those who stated the more general version involving the winding number almost always failed to define the winding number, and even frequently included the symbol without stating what it was. If the winding number was not even named then a mark was lost. Part (c) was well approached with the overall strategy of considering a semi-circle contour in the closed upper half-plane being realised by almost every candidate. The most common errors were incorrect uses of the reverse triangle inequality to obtain an upper bound on $\left((z+b)^2+a^2\right)^{-1}$ on the circular arc of the contour and invoking Jordan's Lemma without verifying that the hypotheses were satisfied. A less frequent but more costly error was the consideration of $\frac{\cos(kz)}{(z+b)^2+a^2}$ rather than $\frac{e^{ikz}}{(z+b)^2+a^2}$ coupled with the failure to realise that the prior of these functions is *not* bounded in the upper half-plane. Part (d) proved to be the hardest part with several candidates failing to complete their answers. Almost every candidate considered a suitable key-hole contour and correctly computed the residues at i and $-i$. Realising that the integrals over both circular arc components of the contour vanished in the limit was common, though there were several poor justifications of these facts involving incorrect uses of the reverse triangle inequality to give an upper bound for $(1+z^2)^{-1}$ and trying to use Jordan's Lemma despite the arcs not being contained within the upper half-plane. There was a split between candidates who realised that defining z^t on $C\setminus[0,\infty)$ required specifying a branch of the complex logarithm and those who did not. Those who did generally correctly calculated the resulting contributions of the integrals over the two straight line segments in the limit and scored highly. Those who did not commonly incorrectly calculated this contribution and then tried to "fudge" the algebraic manipulations to the required answer. A frequent error amongst both sets was a failure to realise that the contribution of these two integrals only becomes a multiple of $\int_0^\infty \frac{x^t}{1+x^2}dx$ in the limit that the radius ep of the small circle around 0 is sent to 0 and the radius R of the large circle is sent to ∞, and not before this limit is taken.

Question 6. Although it is the last question on the paper, it turns out to be relatively popular. The concept part (a), among which half of them can count as book work, received good (but incomplete) answers from the candidates attempt. Candidates may lose points for (a)(iii) and (iv) with justification not to the point. The distribution of marks awarded to part (b) is not linear, a significant number of candidates received good marks for either their working out the conformal mappings or explaining clearly the method combining with clear sketches to determine conformal mappings required. While still many candidates received no or few marks for this part.

Long Options

A3: Rings and Modules

Q1 Almost all candidates answered this question. On specific parts: (a) The degree of the zero polynomial is not 0. Subrings of integral domains might be trivial and so not integral domains. (b) Fields are commutative. (c) Some explained why the proof didn't work rather than why the hypothesis could not be dropped. (d) Everyone completed this well. (e) Some examples were not fields. (f) There were many approaches, some quoting the Fundamental Theorem of Algebra or results from the notes; some giving an explicit uncountable linearly independent set as in the notes; and some giving an explicit countable linearly independent set, though occasionally without justification.

Q2 Many candidates answered this. Few attempts were made to streamline material from the notes leading to long and convoluted answers. On specific parts: (a) Some assumed the division algorithm; a proof was expected. (b) Some used $R[X]$ is a PID iff R is a field which was nice. Some used ED is PID without a proof. (c) There was a lot of quoting of results e.g. that PIDs are UFDs. (d) Some showed that R was a ring though that was not necessary. Many showed R was an ED. Subrings of PIDs need not be PIDs. Some showed that R was Bezout rather than a PID.

Q3 Overall the least familiar question it was not answered by many candidates but was well done when answered. Answers for part (g) included both specific examples of elements not in the image as well as dimension arguments.

A4: Integration

The questions that had been set for a closed book examination were not changed when the examination became open book, although the mark-scheme for Q.3 was adjusted slightly. The main effect of the exam being open book was that the weakest students got higher raw marks than usual, so greatly reducing (perhaps eliminating) the need for scaling of the marks. Each question had really hard parts, so there were few very high marks on the whole paper.

Q.1: In part (a), many students dropped a mark or two. Most candidates got all 4 marks for (b)(i). On the other hand, most candidates got a mark of 0 on (b)(ii) usually because they thought that it was an immediate consequence of (b)(i). In (c)(ii) many candidates thought that a continuous image of a closed set is always a closed set, while others tried to deduce measurability from the formal definition. Generally they were more successful with (c)(iii).

Q.2: This question had virtually no bookwork, so it was no easier in an open book exam. There would probably have been more attempts if the exam had been closed book. A handful of candidates gave good answers to (b), but none gave complete answers to (c).

Q.3: Most candidates got almost all of the first 14 marks, where 7 were available for standard bookwork and 7 for a straightforward application. A fair proportion saw that they should integrate by parts to get the second formula for the derivative, but only a few thought of using that formula to get the existence of the second derivative at most points. Some candidates falsely applied the converse of the differentiation theorem. Nobody gave a complete argument for the non-existence of $f''(0)$.

A5: Topology

The questions were, unfortunately, of unequal difficulty. The second question was somewhat on the too easy side, with a large number of stydents reaching 25/25pts. The 3rd question was challenging for all those who attempted it. In fact, only about 10% of all students attempted the 3rd question. The part about finding an explicit homeomorphism between a certain genus two surface and the "standard" genus two surface was particularly challenging.

A6: Differential Equations 2

Question 1: This was the most popular question and was generally answered quite well. Parts (a) and (b) were bookwork which could be lifted almost verbatim from the lecture notes, so I was looking for precise solutions to get full marks. In part (c), candidates were often confused about how to include the parameter λ and the inhomogeneous boundary conditions in the eigenfunction expansion, and in many cases just ignored them.

Question 2: This was the second most popular question, but was generally found the hardest. Many candidates made heavy weather of part (a), and there were many problems obtaining the correct indicial equations. Given that the wording was not 100% clear, I was lenient on candidates who did not show that there is *precisely* one bounded solution as $x \to 0$ and as $x \to 1$. In part (b), many candidates did not argue persuasively for the "only if" as well as the "if". Although a similar question was on a problem sheet, part (c) was found difficult, especially by weaker candidates, and there were very few totally correct answers. The case $n \neq m$ can be most easily done using Stürm–Liouville theory and exploiting the boundedness of $p_n(x)$. For the case $n = m$, generally insufficient care was taken with the required repeated integration by parts.

Question 3: This was the least popular question, though generally done quite well by those who attempted it. The logarithmic expansion in part (a) proved surprisingly difficult for many (a common fallacy was to try to use the expansion $\log x \sim (x-1) - (x-1)^2/2 + \cdots$ when $x-1$ is not small). In part (b), even when the correct initial-value problem had been derived, there were many problems with the routine calculations needed to calculate y_1. In part (c)(i), I was looking for more than a quotation of a result from the lecture notes. In part (c)(ii), most candidates could see the inductive argument that would lead to the answer, but many were again defeated by standard algebraic manipulations. The boundary layer calculation in part (c)(iii) was generally done well.

A7: Numerical Analysis

Question 1: Every candidate attempted this problem. Given the open-book nature, some of the bookwork problems were probably too basic, as they could be completed by 'copying' from lecture notes. In particular, 1(a) was completely bookwork and (b)(i,ii) nearly so. A common . Many used the error formula derived in class to show the Newton-Cotes rule integrates polynomials of degree n exactly; this is an overkill (as a one-liner using the uniqueness proved in 1(a) would suffice), and would waste time; this was quite visible in those candidates' performance in later problems. (c) requires a combined understanding of orthogonal polynomials and Gauss quadrature, which is where most of the variance in marks lied. About half of the candidates saw the connection between integration and the polynomial coefficients and Gauss quadrature. Only two worked out the first two orthonormal Legendre polynomials including constants to obtain a full mark.

Question 2: Most (about 75%) candidates attempted this problem. (i) solutions split into two types: one based on the lecture notes (best approximation in inner product spaces), and one based on the problem sheets (utilising the QR factorisation). Both merit a full mark of course, but for (ii) the latter approach might have been less straightforward. For (ii), some simply noted a reduction to the form in (i); which is also correct. (b) was bookwork and most obtained full mark. (c) (i) most came up with a valid counterexample, usually 2x2 with one big disk. Some gave a 2x2 answer to (ii) (marks subtracted), and many gave a block diagonal example (not intended but awarded full mark). (d) was intended as a challenging problem. About a third of the candidates did get the right idea of looking at two largest components of the eigenvector.

Question 3: About 25% of the candidates attempted this problem. (a) The SVD appeared in a problem sheet, and will be moved to a core topic from next year. (ii) requires some creativity and many seemed to struggle. (b)(i) Most who attempted Question 3 solved this problem, which is a routine application of companion matrices. (ii) was also mostly successful, though many forgot to mention the endpoints as potential candidates. (iii) was probably the most challenging of all, requiring deep understanding of companion matrices and innovation. To my delight, a few nontheless obtained a full mark.

The performance on Questions 2 and 3 were comparable (and everybody did Q1), so the difficulty was largely the same regardless of the choice of problems attempted.

A8: Probability

See Mathematics and Statistics report.

A9: Statistics

See Mathematics and Statistics report.

A10: Fluids and Waves

50 candidates took the Part A Fluids and Waves Exam. The vast majority answered questions 1 and 2; 13 candidates answered question 3.

The most popular question was question 1:

Nearly everyone obtained full marks on parts 1(a) and 1(b). I marked 1(b)ii out of 2 instead of out of 3, and moved the extra mark to 1(b)iii. Many candidates were unable to derive the expressions for the velocity potential and streamfunction, given in 1(c). 1(d) was well answered. The majority of answers obtained marks above 17, with 7 candidates obtaining either 24 and 25 marks.

The next popular question was question 2:

This wasn't as well aswered as question 1. Many candidates did not prove the result in 2(a) that $\nabla H.\mathbf{u}.0$ using the definition of streamlines. 2(b) was very well answered. However many candidates used complex variables and Blasius's Theorem to compute the force on the wall. Very few candidates used the more direct method of using part 2(a) to integrate the presure in real variables. Nearly all candidates obtained full or nearly full marks for 2(c).

The least popular question was question 3:

This question was the least-well answered. Nearly everyone answered the bookwork parts in 3(a) and (b). Most candidates were able to derive expressions for the dispersion relation in 3(c). Part 3(d) caused a lot of problems, and no-one was able to answer this part correctly

A11: Quantum Theory

The exam was taken by 79 students.

Question 1 was attempted by all students but one. The average grade was 22.5 out of 25. Many students got the top grade. I think the question was well designed, but the fact that was online made the average grade quite high. Part d of the question was solved correctly by most students, and perhaps should have been a bit harder.

Question 2 was attempted by 34 students, and I feel it worked very well. The average grade was 14 out of 25. Almost all students did part a correctly, and then the question got harder, as it should.

Question 3 was attempted by 48 students, and I also feel it worked very well. The average grade was on the higher end (19.4 out of 25) but not abnormally so. Almost all students got the full mark in part a, and solved properly most of part b, while many found part c difficult.

Overall I think the exam worked quite well.

Short Options

ASO: Q1. Number Theory

Part (a) of the question was found to be straightforward by the very great majority of candidates, with almost all attempts being correct or with only minor omissions.

Part (b) of the question had some easier bits, and some harder bits. More candidates than expected found (ii) quite difficult, either misinterpreting the question or making false assertions such as that two infinite sets of primes intersect. Part (iv) was well done by many, though there were also many attempts which did not adequately justify why there cannot be more than 5 nonresidues. Part (v) was only solved by a fairly small percentage of candidates, but there was a good variety of solutions to it, some of them rather nice.

ASO: Q2. Group Theory

Part (a) was generally well done, especially by those candidates who realised the first part was asking for the Class Formula which then could be used in the second part.

In Part (b) the third part proved harder but still many good solutions were produced. [Note, it would have been enough to assume that the subgroup is simple.]

Finally Part (c) was a relatively easy version of a standard question. Candidates were not as comfortable with semi-direct products as one might have hoped. Many also missed that there are three homomorphisms from the cyclic group of order 3 to the cyclic group of order 6, two of which giving rise to isomorphic semi-direct products. Others failed to clearly state that a semi-direct product in which both groups are normal is indeed a direct product.

ASO: Q3. Projective Geometry

The Projective Geometry ASO question was attempted by 10 candidates, many of them submitting excellent answers. The question turned out to be easier than intended, though it is also possible that only relatively strong students attempted this question; the average score on the question was 20.5.

Part (b)(i) caused some difficulty for some; most candidates noticed that the intersection point of the lines given in the hint should be different from the four given points, but some could not clearly articulate why. There were also some gaps in the arguments given in (a)(iii): most students wrote down a correct example of two quadruples that are not projectively equivalent, but some did not correctly argue that this is indeed the case. Other parts were mostly well done.

ASO: Q4. Introduction to Manifolds

14 students attempted the ASO Introduction to Manifolds question, which turned out to be harder than envisaged; the average score was 12.3 and the highest 18. Students began to stumble early on; many missed the fact that in (a) the standard definition needed to be specialised to the codimension 1 case, or failed in (b) to give an argument for why the tangent space is contained in the kernel of the differential of f. In part (c) most students missed the easy proof which fixes one end of a chord between the two graphs and moves the other; moving both ends simultaneously leads to a problem which is harder to analyse, but some students successfully completed this harder task. In the first half of (d), all students missed the fact that to apply the result of (c) one needs to make sure that the submanifolds are disjoint. Finally the second half of (d) leads to long computations unless done smartly; very few students got to the end of this computation, even though many of them sketched the graphs satisfactorily.

ASO: Q5. Integral Transforms

Part (a) was generally well answered. Most candidates were able to calculate the Fourier transforms required for (i) and (ii). For (iii), most candidates correctly identified the limit of the imaginary part of $\tilde{f}(s)$. The most common cause of lost marks for candidates was identifying that the real part of $\tilde{f}(s)$ had integral π, and there was some difficulty in justifying that this tends to $\pi\delta(s)$ as $\epsilon \to 0$. Most candidates who attempted (iv) recognised that they could write 1 and $\delta(x)$ in terms of the sum of two Heaviside functions / derivative of a Heaviside function, although some candidates lost marks for using methods that did not use (iii) or for simply stating the relevant Fourier transforms from memory.

Part (b)(i) was well answered, and in (ii) most candidates were able to solve the differential equation for y. Relatively few candidates identified that y was bounded for $n = 1, 2$ as $x \to \infty$. Part (iii) caused the most problems in question (b), with candidates attempting a range of methods to calcuate the Laplace transform. While some candidates recognised the parallels with (i) and rewrote the equation as $\frac{d^n G_n}{dx^n} = g(x)$, many students instead calculated the Laplace transform iteratively. Most candidates who were able to calculate the Laplace transform of G_n recognised that they could use the convolution theorem to reach the answer.

Many candidates either did not attempt (c) or were unable to progress beyond writing the formula for the Fourier transform of $\cos(x)$. Candidates who recognised that $h(x)$ was even generally had the most success with calculating the integral, although many candidates who did not use that simplification ultimately reached the correct answer. Those students who successfully calculated the integral generally recognised that I_1 and I_2 could be solved by using the inversion formula with $x = 0$ and $x = 1$ respectively.

ASO: Q6. Calculus of Variations

Overall the Calculus of Variations question worked reasonably well, despite the complications caused by the move to open book format. The open book nature of the exam meant that there was probably a narrower range of scores than one would otherwise expect since the earlier parts were close to the notes, but the question was still able to distinguish between students.

The early parts (a) and (b) of the question were close to content covered in lectures, and so unsurprisingly were generally answered well. A few marks were lost for not appreciating that this was a restricted optimisation problem or that it was important that the test function was arbitrary beyond a couple of restrictions. In part (c) essentially no student noticed the subtle point that $y(x)$ =constant is also a solution to the differential equation, but otherwise the calculations were generally done well. The final harder part (d) of the question attracted a variety of responses, with some candidates failing to make progress, some answering in part, and some giving essentially complete answers.

ASO: Q7. Graph Theory

The graph theory question was fairly popular, with 28 attempts. In the assessor's view, the question was a successful one. It was based on the theory of matchings, which is a central part of the course. Every candidate could demonstrate at least some basic understanding of matchings, but very few were able to give perfect or near-perfect solutions to every part of the question.

All candidates could make a start with the question and find the required matchings in the given graph. Almost all candidates could use Hall's theorem to show that a k-regular bipartite graph is a union of k perfect matchings. However, the remaining parts of the question were more discriminating. Many candidates were unable to prove that a bipartite graph with every vertex degree at most k is a union of k matchings. The most natural proof of this to enlarge the graph by adding edges and possibly vertices until every vertex degree is k and then to apply the previous part. The final part of the question, on matchings in complete graphs, proved to be fairly challenging. Quite a few candidates were able to show that K_n is not a union of $n-1$ matchings when n is odd, by counting the number of edges in each matching. However, to show that K_n is a union of n matchings required a direct construction, which eluded many. Both in this subpart and in (b)(iii), there were quite a few failed attempts at inductive proofs.

ASO: Q8. Special Relativity

This problem had three parts.

- Part (a) was bookwork, and was (unsurprisingly) answered very well, though there was a fine point regarding the positivity of the pseudo-norm $g(U+V, U+V)$ which required noting that the sum of future-pointing timelike vectors is future-pointing timelike.
- Part (b) was a computation in relativistic kinematics and was answered well up to algebraic inaccuracies.
- Part (c) was a problem dealing with an observer undergoing piecewise-constant acceleration. While part (i) was standard, parts (ii)–(iv) were trickier. An important mistake

to be avoided was changing the sign of the *three-acceleration* rather than the *four-acceleration*, which would lead to inconsistent results since the (faulty) four-acceleration would then not be pseudo-orthogonal to the instantaneous four-velocity.

ASO: Q9. Modelling in Mathematical Biology

Marks were very high. They were only really lost where students reverted to thinking of stability of differential equations ($re\lambda < 0$) rather than difference equations ($\lambda < 1$); of course there was the odd algebraic error.

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